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Original Contribution

THE IMPACT OF REFLECTION ON MATHEMATICS PROBLEM SOLVING

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ABSTRACT
Reflection reveals itself in the act of understanding. The notion of reflection is widely spread in variety of activities. The problem of implementation of the reflection in the process of mathematics has recently become quite significant since the deeper amalgamation of the basic psychological aspects in the modern people life, and the invading globalization, new information technologies and the sophistication in the communications and interactions.
In the article we give a short theoretical description of the notion of reflection. Later on we study a specific mathematical problem with an emphasis on reflection rather than on the solution. We also discuss the importance of this approach in the preparation of the Mathematics Olympiad team in Bulgaria.

Key words: reflection, problem solving, mathematics olympiads in Bulgaria

THEORETICAL FOUNDATIONS OF THE REFLECTION

In satisfaction the demands of the contemporary mathematics education, reflection is the leading mechanism for stimulating students intellectual capabilities. The growing interest of reflection as a part of mathematics education is a direct consequence of the belief that reflection in fact governs the process of education towards the very student personality. In that way the psychological aspect of teaching becomes more transparent. By the reflection teacher becomes a real facilitator of knowledge and the act of problem solving turns into an act of learning. Finally, pedagogical technologies orientated towards development of the studying abilities consist of two major components -- technology of the teacher's activity and technology of the student's activity which together activate the reflective and innovative moments in thinking, communication and pedagogical activity in general.

Reflection pushes ahead the general cognitive abilities such as intelligence, creativity, etc. On the other hand reflection demands a particular environment in stimulating innovations [1]. Reflection is also one of the mechanisms of converting teaching into an optimal correspondence between the study content and the subjective experience. In [2] reflection is defined in much broader sense as self-understanding and understanding of the others; as self-evaluation and evaluation of the others; as self-interpretation and interpretation of the others. Some authors assume that reflection is a subjective form of a psychological existence and that it is a foundation for the personal development and a necessary component of the general cognitive abilities [3]. According to V. Davidov there are two general ways of thinking -- theoretical and empirical and in the process of learning how to read, write, compute, etc. (for example the abilities to sing or recite) among kids, there are elements of reflection and analysis [4]. Moreover, it is possible to achieve a generation of reflection in the very early childhood [1].

Contemporary research in mathematics education deals mainly with the pedagogical factors in the process of creating and developing reflection. Pedagogical reflection

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may be considered as a skill because it allows specific and unrepeatable situations which emerge in education to be correctly defined. On the other hand, it may also be considered as ability because its development depends, to some extent, on some genetically conditioned prerequisites.

Generally, one can say that reflection is one of the most valuable tools for the student intellectual stimulation in mathematics education. Mastering of the abstract mathematical knowledge is deeply connected to reflection. The main reasons to consider the reflexive aspects in the act of problem solving are:

- necessity to shape the reflection in the university as well as in the high-school education;
- one of the basic characteristics of the reflexive studying is that it is orientated towards some of the characteristics of the intellect such as the problem solving;
- the unique role of the math teacher as a facilitator of knowledge and abilities to solve problems. Development of the reflection in the act of solving mathematics problems is the main goal (one of the most important) in the future mathematics teachers' education.

The main aspect of this article is not the reflection in general but rather the description of some phenomena in its appearances. Therefore we shall adapt a definition for the reflection in the act of problem solving which is a narrow one but which possesses a strong explicable and prognostic function:

Definition: Reflection in the act of problem solving is the projection of our samples of thinking and acting. This projection is a continuous (throughout the entire process of the problem solving) introspection which governs the "insight".

Majority of those who solve problems do not have the reflection skill. In our experimental groups of mathematics teacher we set the requirement to pay attention of their thinking in different series of problems. Despite that requirement even those who successfully solved their problems and showed good mathematics skills, were hardly able to track down their own paths of thinking. It was obvious there was no "inner spectrum" or "inner eye" developed. Those who solved their problems were completely careless about how they solved these problems. The reflective summary turned out to be a tedious task even for problems that admit different approaches and are not so difficult to tackle.

Development of the prognostic functions of the methods and ideas of solving and binding them with the heuristic techniques and specifics of the process of problem solving is a good basis for a full spreading of the reflection.

First, author's analysis shows that he has developed his reflective skills for a period of twenty years with persistent mastering. It turned out that the development of the skills in mathematics problems solving is quite important in terms of the intellectual development. It is necessary to point out the relation education--heuristic/algorithmic--reflection—creativity in the act of problem solving. [1]

It is a well-known fact that the most common scheme of how the process of problem solving goes on is separated in four stages. Reflection appears at the fourth stage and creates a specific environment for binding some operational schemes in verbal forms. That is probably the most important part at the fourth stage in solving a mathematical problem. In the next two sections we shall describe the mathematics and informatics competitions scene in Bulgaria and shall solve a specific Olympiad problem supplying the necessary reflective summary.

MATHEMATICS AND INFORMATICS COMPETITIONS IN BULGARIA.

Mathematics and Informatics competitions are among the best tools to identify and develop the intellectual abilities of young people. Main goals in the activities of preparation and self-preparation for the mathematics and informatics olympiads are the development of the environment for a mathematics creativity, deepening and broadening of skills for capturing knowledge, and finally the developing of the mathematics activities and increasing capabilities of the students participating in the olympiads [5]. The preparation for these events involves quite a number of secondary students and teachers and increases the level of learning and teaching mathematics and informatics in particular. The competitions also provide (relatively rare) opportunity to focus the public attention on the role of mathematics and information sciences in the contemporary society.

Last, but not least, talented youngsters get encouraged to pursue career in science. This keeps up the health of mathematical and information sciences in the country and attracts "fresh blood" to other branches of science as well.

In the area of such competitions Bulgaria has long traditions. The National Mathematics Olympiad is now (end of 2009) 58 years old. The National Olympiad in Informatics started in 1985. Even before the beginning of these olympiads, competitions on local and/or regional level were popular. Bulgaria is a co-founder and regular participant in the International Mathematics Olympiad (IMO) since its commencement in Romania in 1959. The IMO is a competition for high school students. Although students compete, unofficial country rankings are obtained by adding the individual students' scores. Bulgaria is almost always among four top-scored countries. The IMO results from 1995 till 2004 can be seen in the table below. (The number in the brackets is the country's unofficial score and the number above is the country's rank).

YEAR / Country	2004	2003	2002	2001	2000	1999	1998	1997	1996	1995
CHINA	1 (220)	2 (211)	1 (212)	1 (225)	1 (218)	1 (182)	–	1 (220)	6 (163)	1 (236)
RUSSIA	3 (205)	5 (167)	2 (204)	2 (196)	2 (215)	1 (182)	6 (175)	4 (202)	4 (162)	3 (227)
USA	2 (212)	3 (188)	3 (171)	2 (196)	3 (184)	10 (150)	3 (186)	4 (202)	2 (185)	11 (178)
BULGARIA	5 (194)	1 (227)	4 (167)	4 (185)	5 (169)	5 (170)	2 (195)	7 (191)	11 (136)	6 (207)
VIETNAM	4 (196)	4 (172)	5 (166)	10 (139)	5 (169)	3 (177)	9 (158)	10 (183)	7 (155)	4 (220)

Table 1. *Unofficial Country rankings -- IMO*

Bulgaria is also originator and the first host of the International Olympiad in Informatics (IOI) which was held in Pravetz, Bulgaria in 1989. The work with talented secondary school students is boosted also by the regional competition called Balkan Mathematics Olympiad which is regularly taken place since 1984.

The preparation for the competitions is held in groups and clubs which exist not only is special math and science schools but in many other schools as well. Usually students start to participate in competitions since the first year of the high school (age 10-11). Students compete in problem solving. They receive 3-4 problems which they should solve for approximately 4-4.5 hours. Recently, multiple choice tests are gradually getting place in Bulgaria.

The most important mathematics competitions on national level in Bulgaria are:

- The National Mathematics Olympiad;
- The National Informatics Olympiad;
- The Winter Mathematics Competition;
- The Spring Mathematics Competition.

There are also many local competitions. The national mathematics and informatics competitions and the preparation for the international competitions are organized by the Ministry of Education and by the Union of Bulgarian Mathematicians (UMB). The

Ministry of Education provides the also responsible for significant part of the financial support of the competitions. The UBM is responsible for the academic part. A special committee dealing with the scientific organization of the competitions has been found at the UBM. The members of this committee are mainly from the Department of Mathematics at the Sofia University and scientists from the Institute of Mathematics and Informatics of the Bulgarian Academy of Science. This committee makes the problem selection for the above mentioned competitions, organizes the checking and marking of the students' papers, organizes seminars and workshops for teachers who are leaders of math clubs and publishes materials for these seminars, etc.

In the next section we will pick a specific mathematics Olympiad problem and will try to focus on the reflection in the act of problem solving rather than on finding a solution to the problem.

1. One particular example

We shall start with one math problem which was a part of the National Mathematics Olympiad some 30-35 years ago [6]. First we shall present the solution of those who suggested the problem and then the solution of the second author of this article with an emphasis on the reflection.

Problem. There are 20 different natural numbers $a_i,\ i=1,2,...,20$ less than 70 given. Prove that there exist at least four equal differences $a_i - a_j,\ i > j$ among these numbers.

<u>Author's solution</u>: Let's order the numbers given: $a_1 < a_2 < ... < a_{20}$. We consider the consecutive differences $a_2 - a_1,\ a_3 - a_2,\ a_4 - a_3, ...,\ a_{20} - a_{19}$ and administrative base for the competitions. It is assume there are not four differences among these which are equal.

Therefore the smallest among these differences can be encountered at most three times and since $a_i \neq a_j,\ i \neq j,$ this difference is greater or equal to 1. The next difference can be encountered at most three times as well and this difference is greater or equal to 2 and so on. The sum of all possible differences is

$$(a_2 - a_1) + (a_3 - a_2) + ... + (a_{20} - a_{19}) = a_{20} - a_1.$$

Therefore

$$a_{20} - a_1 \geq 3\cdot1 + 3\cdot2 + 3\cdot3 + 3\cdot4 + 3\cdot5 + 3\cdot6 + 7 = 70$$

But this is not possible since $a_1, a_2, ..., a_{20} > 0$ and $a_i < 70,\ i = 1,2,...,20.$ That proves the statement.

<u>Our solution</u>: In our solution we shall map the set of all differences to the set of their values. We hope this solution will reveal the importance of the reflective summary in the act of problem solving.

Let Δ be the set of all differences $a_i - a_j,\ i, j = 1,2,...,20,\ i > j.$ We have that $|\Delta| = 19 + 18 + ... + 1 = 190.$ Since $a_i \in N,\ a_i < 70,$ it follows that $a_i - a_j \in \{1,2,3,...,68\},\ i > j,$ i. e. the set of all integers from 1 to 68. It follows immediately that there exist at least two equal differences since $2 \cdot 68 < 190.$ We put these two equal differences aside. The set of all values of $a_i - a_j$ can be divided into four groups M_1, M_2, M_3, M_4 modulo 4, i.e.

$$M_1 = \{1,5,9,...,65\},\ M_2 = \{2,6,10,...,66\},\ M_3 = \{3,7,11,...,67\},\ M_4 = \{4,8,12,...,68\}.$$

We have that $|M_1| = |M_2| = |M_3| = |M_4| = 17$ and each difference $a_i - a_j,\ i > j$ belongs to some $M_i,\ i = 1,2,3,4.$ On the other hand

$$3|M_1| + 3|M_2| + 3|M_3| + 2|M_4| + 1 = |\Delta| - 2.$$

There are two principal possibilities now.

This last difference to belong to M_1, M_2 or M_3. If this is the case, it immediately follows that there exist at least four differences in one of the sets M_1, M_2 or M_3 or this last difference to belong to M_4. In this case we take the two equal differences we have already put aside. If they belong to M_1, M_2 or M_3, there exist at least five equal differences and if they belong to M_4, there exist at least four equal differences. Thus, in any case there exist always at least 4 equal differences and this solves our problem.

CONCLUSION

At the end, we would like to distinguish the following functions of the reflection in solving mathematics problems which can be used in mathematics education:

- The skills and knowledge and their generalized forms in the act of problem solving are direct consequences of the reflection;
- The main product of the reflection is the description of the search of a solution and the educational solutions in particular. The second solution of the problem above is an educational solution and the first is not;
- Reflection contributes immensely in the sequence: education – anticipation – algorithmic (heuristic – reflection – creativity;
- One goof reflective summary in the act of problem solving is an opportunity to evaluate the developing functions of the problem and to construct different technological variants of series of problems;
- Pedagogical activity of the mathematics teacher is the most reflective one. Mathematics teacher has a key role as a living facilitator of knowledge and skills.

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