BIOPHYSICAL MODELLING OF VISCOELASTIC DEFORMATIONS OF CATTLE SKIN IN VIVO

Penka Dulgerova*

Department of “Informatics, Mathematics and Physics”, Trakia University, Stara Zagora, Bulgaria

ABSTRACT

The biophysical modelling of cattle skin deformation processes in vivo is presented in this paper. The properties of cattle skin, as viscoelastic material, are described. The choice of the appropriate biophysical model of the cattle skin deformation in vivo is discussed. A differential equation, describing the behaviour of cattle skin under the tension, is composed. The interpretation of the solution of this equation is made. The physical meaning of the “time-constant” is discussed. The possibility of obtaining indirect information of the elastin-collagen network structure is presented.

Key words: biophysical modelling, cattle skin deformation, time-constant

INTRODUCTION

The cattle skin is a typical viscoelastic material according to its behaviour under mechanical deformations. It possesses both elasticity and viscosity. These skin properties are determined from the proteins collagen and elastin in the derma, which are typical biopolymers. The collagen macromolecules are long polypeptide chains. There are interactions and connections between the molecules as well as between the separate fragments within the molecule. These interactions fasten and stabilise the collagen structure and determine its physical-mechanical properties. The collagen macromolecules are joined in macrostructures above the molecule – prothofibrilles, fibrilles, fibres and sheaves of fibres. They are relatively strong and have the elasticity module $10^8$ Pa (1-7).

According to contemporary notion the polypeptide collagen chains have a spiral configuration. Three polypeptide chains, each a counterclockwise spiral, are screwed together around the common axis, forming a clockwise screwed spiral. The opposite torsion resists lengthwise deformation and contributes to the resistance of the molecule against stretching. This is the main factor determining mechanical functions and properties of the collagen of the skin as a whole (1,5).

The elastin is a fibrous protein in skin elastin fibres. These fibres possess extraordinary elasticity, which explains their name. They are elastomers according to their behaviour – they have small elastic module and restore their initial size even after significant stretching. The structures in the tissues, with elastin as their basic component, are called elastic and are interesting because of their specific biomechanical properties. The elastin has amorphous network structure with a small extent of arrangement. This structure explains the high elasticity of its fibres, which are built from long flexible disordered chains of molecules. The orientation of the molecules occurs when the fibres are stretched (8, 9).

The elastin fibres are interlaced between the collagen fibres and a part of them are spirally screwed around them. These complex fibres form the dense network, which is situated mainly in an upper part of the derma.

The third type of the fibre protein – reticulin – is in small amount in the derma – around 0,38 % of the derma dry material. That is why it does not play important role in the skin mechanical properties (1).

The complex network of interlaced collagen and elastin filaments is “immersed” in a high viscosity jelly material (1,2,3). This material, as well as the interaction between
polypeptide chains of the protein molecules, slows down the mechanical processes in the skin; this can be regarded as manifest of viscosity. When mechanical tension is applied on some certain skin area the skin becomes deformed as a result of reorientation and extension of the collagen and elastin filaments. When subjected to heavier tensions under these deformation, changes at the molecular level occur which essentially consist of a change in the spatial configuration of the polypeptide chains of the collagen and elastin.

After the termination of the external tension the elastin filaments show some extraordinary elasticity. They restore their initial condition and engage the collagen network they interlace with. The deformation and restoration processes are not instant, but occur with slow motion because of the viscosity of the material. The specifics of the skin deformation provide information about its direct theoretical and indirect practical nature of its structure - for the protein fibril network quality.

The importance of the problem determines the researcher’s focus on the investigation of the cattle skin deformation in vivo.

MODELLING
The deformation and restoration properties of the cattle skin can be examined as a function of time by biophysical modelling (10, 11).

The choice of the appropriate biophysical modelling of the cattle skin deformation in vivo is determined by the skin structural specifics and the extent of the skin mechanical tension in vivo in the research process. The collagen molecules refer to the so-called “sewed” polymers, whose typical feature is not “flowing”. In the process of skin deformation in vivo pure phases of “elastic deformation” and “flowing” are not observable.

The most suitable basic model for the cattle skin deformation in vivo is the Kelvin-Foight model (Fig.1), applicable in cases of modelling of “sewed” polymers (12). The Kelvin-Foight model consists of parallel connected elastic element (spring) and element with viscosity (damper).

The mechanical tension \( \sigma \), deforming the model can be regarded as a sum of two tensions, \( \sigma_1 \) and \( \sigma_2 \), acting correspondingly on the spring and the damper:

\[ \sigma = \sigma_1 + \sigma_2 \]  

The tension \( \sigma_1 \) can be expressed by Hook’s law:

\[ \sigma_1 = E \varepsilon \]  

Where \( E \), called module of elastic deformation (Young’s modulus), is a constant, characterising elastic properties of the material and \( \varepsilon \) is a relative deformation of the elastic body, invoked by the applied tension \( \sigma \).

The tension \( \sigma_2 \) can be expressed from Newton’s law of the viscosity:

\[ \sigma_2 = \eta \frac{d\varepsilon}{dt} \]  

Where \( \eta \) is the viscosity of the viscous element, and \( \frac{d\varepsilon}{dt} \) is a velocity of its deformation (first derivative from the deformation to the time).

By replacement of /3/ and /2/ in /1/ the resulting equation is:

\[ \sigma = E \varepsilon + \eta \frac{d\varepsilon}{dt} \]  

Expressing \( \varepsilon \) from the above we have:

\[ \varepsilon = \frac{\sigma}{E} - \frac{\eta}{E} \frac{d\varepsilon}{dt} \]  

The deformation \( \varepsilon \) of the model under the tension \( \sigma \) is expressed by equation /5/. If the tension is interrupted, the body starts to restore its initial condition (not deformed). To find the law of the decreased deformation, we transform equation /5/ replacing \( \frac{\eta}{E} \) with \( \tau \).
And we have \( \varepsilon = \frac{\sigma}{E} - \tau \frac{d\varepsilon}{dt} \) \( /6/ \)

When the tension is interrupted its value is zero and equation \( /5/ \) becomes:
\[ \varepsilon = \tau \frac{d\varepsilon}{dt} \] \( /7/ \)
The solution of this equation is:
\[ \varepsilon = \varepsilon_0 e^{-\frac{t}{\tau}} \] \( /8/ \)

In the equation \( \varepsilon_0 \) is the value of the deformation at the moment of tension interruption and \( \varepsilon \) is the value of the decreasing deformation at the moment in time \( t \) after the tension interruption.

As the equation \( /8/ \) shows, after the tension interruption the system moves back to the initial state in an exponential order.

To show the physical meaning of \( \tau \) we replace \( \tau \) with \( t \) in \( /8/ \) and have:
\[ \varepsilon = \varepsilon_0 e^{-t/\tau} \]
and
\[ \varepsilon = \frac{\varepsilon_0}{e} \] \( /9/ \)

Equation \( /9/ \) shows that, at a particular moment \( t \), the deformation \( \varepsilon \) has value \( \varepsilon \) times less than \( \varepsilon_0 \). Hence \( \tau \) is a constant, equal to the time in which the deformation decreases 2.72 times. The value \( \tau \) is called “time-constant” – an important characteristic of the viscoelastic material. The value \( \tau \) is a more suitable characteristic of the deformation diminution than the full time of reaching to the equilibrium state. The value \( \tau \) is similar to the semi decay period as a characteristic of radioactive elements stability.

The time-constant \( \tau \) is important characteristic of the cattle skin viscosity, also. It is composed by the viscosity \( \eta \) and elasticity module \( E \). The higher the viscosity, the slower are the deformation processes, and the bigger is \( \tau \).

The measurement of the cattle skin \( \tau \) gives an indirect information about the strength and the elasticity of the dermal network of the proteins collagen and elastin, which determine the skin mechanical properties. The results can be interpreted biologically and physically.

REFERENCES