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## MODELING NETWORK FLOW BY EXCEL SOLVER

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### ABSTRACT

This paper presents two modeling approaches for solving the max flow problem and Min-Cut Problems. We have illustrated it using a numerical example and formulated two spreadsheets models using the popular spreadsheet package Microsoft Excel Solver. In the first approach, we formulated an algebraic model, then transferred it to a spreadsheet. In the second approach, we formulated a spreadsheet model directly by using spreadsheet modeling techniques.

**Key words:** Excel Solver, max flow problem, Min-cut problems, operations research education, mathematical model, binary variables.

The network flow models are a special case of the more general linear models. The class of network flow models includes such problems as the transportation problem, the assignment problem, the shortest path problem, the maximum flow problem, the pure minimum cost flow problem, and the generalized minimum cost flow problem. It is an important class because many aspects of actual situations are readily recognized as networks and the representation of the model is much more compact than the general linear program. When a situation can be entirely modeled as a network, very efficient algorithms exist for the solution of the optimization problem, many times more efficient than linear programming in the utilization of computer time and space resources. Network models are constructed by the Math Programming add-in and may be solved by either the Excel Solver, Jensen LP/IP Solver or the Jensen Network Solver.

### 1. The max flow problem

This task shows how to find the maximum throughput possible for a network or partition

of the network (roads, water mains, etc.) Consider a graph with a set of vertices  $V$ , a set of edges  $E$ , and two distinguished nodes 0 and  $F$ . Each edge has an associated capacity  $u_{ij}$  but no associated cost. We will assume that there is no edge from  $F$  to 0. In a max-flow problem, the goal is to maximize the total flow from 0 to

$F$ . In our formulation, nodes do not produce or consume flow, but rather, we introduce an auxiliary edge  $(F, 0)$  with no capacity limit and aim to maximize flow along this edge. By doing so, we indirectly maximize flow from 0 to  $F$  via the edges in  $E$ . The max-flow problem is given by maximize  $F$ .

We can model the max flow problem as a linear model.

Variables: Set up one variable  $x_{uv}$  for each edge  $(u, v)$ . Let's just represent the positive flow since

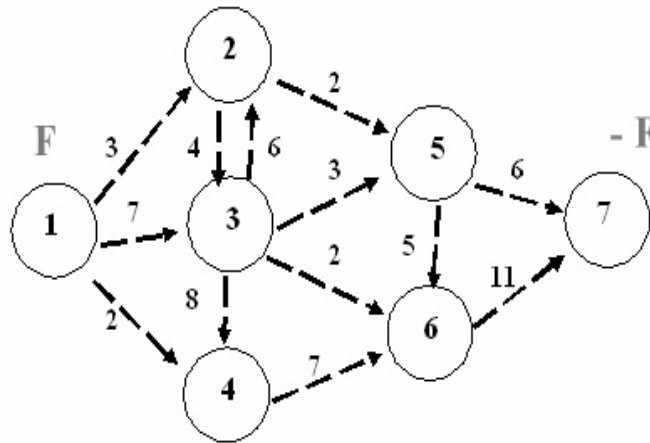
it will be a little easier with fewer constraints.

Constraints:

– For all edges  $(u, v)$ ,  $0 \leq x_{uv} \leq c(u, v)$ . (capacity constraints)

– For all  $v \in \{s, t\}$ ,  $\sum_u x_{uv} = \sum_w x_{vw}$ . (flow conservation)

**Example:** The picture shows such a network with 7 nodes and 13 edges. For each of the edges was recorded and the maximum throughput. Network has one peculiarity - it has a beginning and an end to this essential and is used in the play of the task. We assume that the node 1 (top) is available  $F$ , at 7 (end) is available- $F$ , but in all other nodes is zero availability. Thus the network is balanced, and  $F$  is the number throughputs, which we will seek maximum. The mathematical model is:



$$\begin{aligned} \max F \\ x_{12} + x_{13} + x_{14} &= F \\ x_{23} + x_{25} - x_{12} - x_{32} &= 0 \\ x_{32} + x_{34} + x_{35} + x_{36} - x_{13} - x_{23} &= 0 \\ x_{46} - x_{14} - x_{34} &= 0 \\ x_{56} + x_{57} - x_{25} - x_{35} &= 0 \\ x_{67} - x_{36} - x_{46} - x_{56} &= 0 \\ -x_{57} - x_{67} &= -F \\ x_{12} \leq 3, x_{13} \leq 7, x_{14} \leq 2, x_{23} \leq 4, x_{25} \leq 2 \\ x_{32} \leq 6, x_{34} \leq 8, x_{35} \leq 3, x_{36} \leq 2, x_{46} \leq 7 \\ x_{56} \leq 5, x_{57} \leq 6, x_{67} \leq 11, x_{ij}, F \geq 0 \end{aligned}$$

We will start with solving a max flow problem by using Excel Solver. You can find details about how to use Excel Solver on the Internet. First of all, we can introduced the dates of the mathematical model in Excel spreadsheet. (Fig. 1). All formulas should be entered into Excel spreadsheet. Next step is to open the Solver dialog window. The Set target

cell box is filled with \$O\$3. Select “Max” in “Equal to” row. Then click By changing cells box and select B3-O3. Next step is to click in ”Subject to the Constrains” and add the constrains. Then we click “Solve” button on the right top corner. After a few seconds, you will get solution in B3-O3. You can notice the value in O3 is max flow value.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1		names																
2		x12	x13	x14	x23	x25	x32	x34	x35	x36	x46	x56	x57	x67	F			
3		3	7	2	3	0	0	5	3	2	7	0	3	9	12			
4		Constraints																
5	U1	1	1	1	0	0	0	0	0	0	0	0	0	0	-1	rayno	0	0
6	U2	-1	0	0	1	1	-1	0	0	0	0	0	0	0	0		0	0
7	U3	0	-1	0	-1	0	1	1	1	1	0	0	0	0	0		0	0
8	U4	0	0	-1	0	0	0	-1	0	0	1	0	0	0	0		0	0
9	U5	0	0	0	0	-1	0	0	-1	0	0	1	1	0	0		0	0
10	U6	0	0	0	0	0	0	0	0	-1	-1	-1	0	1	0		0	0
11	U7	0	0	0	0	0	0	0	0	0	0	0	-1	-1	1		0	0
12		1	0	0	0	0	0	0	0	0	0	0	0	0	0	<=	3	3
13		0	1	0	0	0	0	0	0	0	0	0	0	0	0	<=	7	7
14		0	0	1	0	0	0	0	0	0	0	0	0	0	0	<=	2	2
15		0	0	0	1	0	0	0	0	0	0	0	0	0	0	<=	4	3
16		0	0	0	0	1	0	0	0	0	0	0	0	0	0	<=	2	0
17		0	0	0	0	0	1	0	0	0	0	0	0	0	0	<=	6	0
18		0	0	0	0	0	0	1	0	0	0	0	0	0	0	<=	8	5
19		0	0	0	0	0	0	0	1	0	0	0	0	0	0	<=	3	3
20		0	0	0	0	0	0	0	0	1	0	0	0	0	0	<=	2	2
21		0	0	0	0	0	0	0	0	0	1	0	0	0	0	<=	7	7
22		0	0	0	0	0	0	0	0	0	0	1	0	0	0	<=	5	0
23		0	0	0	0	0	0	0	0	0	0	0	1	0	0	<=	6	3
24		0	0	0	0	0	0	0	0	0	0	0	0	1	0	<=	11	9
25		0	0	0	0	0	0	0	0	0	0	0	0	0	1	>=	0	12

Fig. 1. Modeling in Excel

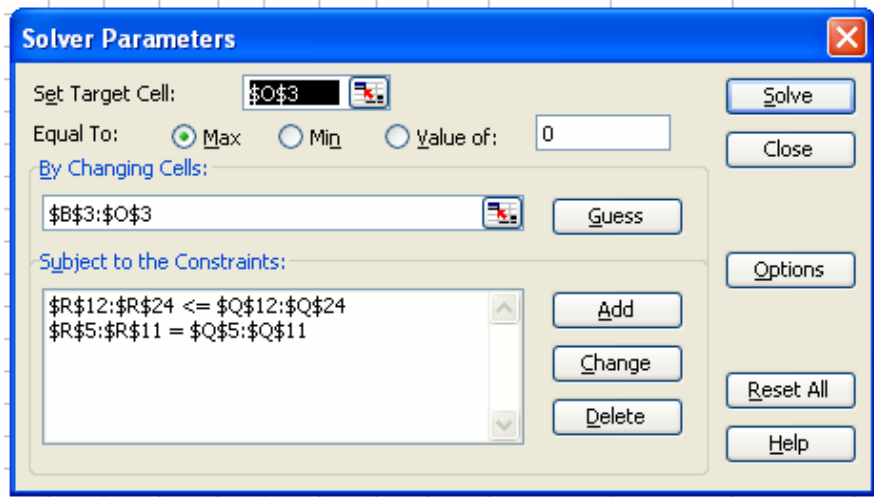


Fig. 2. Solution by Excel solver:

The program makes the next analysis (Fig. 3)

Cell	Name	Original Value	Final Value
\$O\$22	F	12	12

Cell	Name	Original Value	Final Value
\$B\$22	x12	3	3
\$C\$22	x13	7	7
\$D\$22	x14	2	2
\$E\$22	x23	1	3
\$F\$22	x25	2	0
\$G\$22	x32	0	0
\$H\$22	x34	5	5
\$I\$22	x35	1	3
\$J\$22	x36	2	2
\$K\$22	x46	7	7
\$L\$22	x56	0	0
\$M\$22	x57	3	3
\$N\$22	x67	9	9
\$O\$22	F	12	12

Cell	Name	Cell Value	Formula	Status	Slack
\$R\$24	U1	= 0	\$R\$24=\$Q\$24	Not Binding	0
\$R\$25	U2	= 0	\$R\$25=\$Q\$25	Not Binding	0
\$R\$26	U3	= 0	\$R\$26=\$Q\$26	Not Binding	0
\$R\$27	U4	= 0	\$R\$27=\$Q\$27	Not Binding	0
\$R\$28	U5	= 0	\$R\$28=\$Q\$28	Not Binding	0
\$R\$29	U6	= 0	\$R\$29=\$Q\$29	Not Binding	0
\$R\$30	U7	= 0	\$R\$30=\$Q\$30	Not Binding	0
\$R\$31	<=	3	\$R\$31<=\$Q\$31	Binding	0
\$R\$32	<=	7	\$R\$32<=\$Q\$32	Binding	0
\$R\$33	<=	2	\$R\$33<=\$Q\$33	Binding	0
\$R\$34	<=	3	\$R\$34<=\$Q\$34	Not Binding	1
\$R\$35	<=	0	\$R\$35<=\$Q\$35	Not Binding	2

\$R\$36	<=	0	\$R\$36<=\$Q\$36	Not Binding	6
\$R\$37	<=	5	\$R\$37<=\$Q\$37	Not Binding	3
\$R\$38	<=	3	\$R\$38<=\$Q\$38	Binding	0
\$R\$39	<=	2	\$R\$39<=\$Q\$39	Binding	0
\$R\$40	<=	7	\$R\$40<=\$Q\$40	Binding	0
\$R\$41	<=	0	\$R\$41<=\$Q\$41	Not Binding	5
\$R\$42	<=	3	\$R\$42<=\$Q\$42	Not Binding	3
\$R\$43	<=	9	\$R\$43<=\$Q\$43	Not Binding	2

Fig. 3. Answer Report

## 2. Min-Cut Problems

An interesting related problem is the min-cut problem. The objective in the min-cut problem is to find a cut that such that the capacity for flows from  $V_s$  to  $V_d$  is minimized. One way to represent a choice of partition involves assigning a binary variable to each node. In particular, for  $V_s$   $p_s = 1$ , for  $V_d$   $p_d = -1$  and each another node  $i - V_i$ , value  $p_i = 0$ .

If the numbers make sense in edges of lengths similar to formulate the problem of finding the shortest (longest) path from start to end nodes. To "proceed" flow in such a network, the

starting node to put a stock 1, in cash last node is -1 in all other nodes is zero availability - thus the network is balanced. For a description of the mathematical model using binary variables  $X_{ij}$ . If the decision variable is 1, click on the corresponding arc of interest to enter into our itinerary. Includes functional model (sum of the products of "length" of arcs in the corresponding  $X_{ij}$  for all arcs) which sought a minimum or maximum and balance equations, written as a general rule.

Solving the problem is done with the program Excel solver, adding an extra constrain for binary variables. **Figure 4** shows the constrain is added to Excel solver.

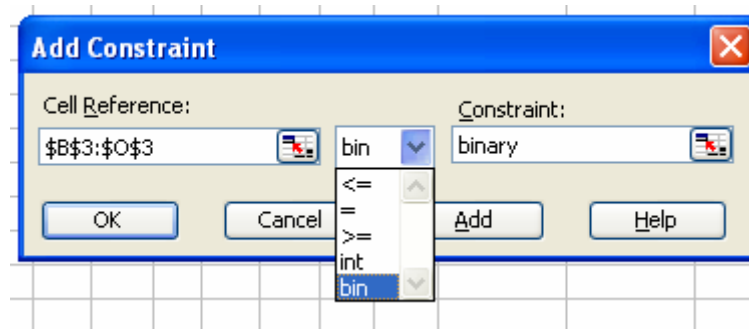


Fig. 4. Binary variables

Excel is a powerful application. With additional Add-ons, it can be used to solve very complicated problem. Excel Solver can use to solve simple non-linear problem and extend the method to solve neural network problem.

## REFERENCE

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