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Original Contribution

ANALYTICAL STUDY OF LINEAR INSTABILITY OF AN ANNULAR LIQUID SHEET EXPOSED TO GAS FLOW

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ABSTRACT

A linear instability analysis of an inviscid annular liquid sheet emanating from an atomizer subjected to inner and outer swirling air streams has been carried out. The dimensionless dispersion equation that governs the instability is derived. The dispersion equation solved by Numerical method to investigate the effects of the liquid–gas swirl orientation on the maximum growth rate and its corresponding unstable wave number that it produces the finest droplets.

Keywords: Annular liquid sheet, Atomization, Linear instability, Swirl.

INTRODUCTION

Liquid atomization is of importance in numerous applications such as fuel injection in engines, gas turbine engines, industrial furnaces, agricultural sprays [1][1].

Pressure swirl atomizers are being recognized as ideal atomizers for the direct injection spark ignition (DISI) engines or gasoline direct injection (GDI) engines because they generate a fine fuel spray with moderate injection pressure. The advantageous characteristics of pressure swirl atomizer include simplicity of construction, ease of manufacture even in small size, reliability, good atomization quality, low clogging tendencies, and low pumping power requirements [2].

The stability of liquid jets and sheets has received much attention since the classical studies of Rayleigh and Squire. For authoritative reviews of liquid sheet and jet instability and breakup, readers are referred to a recent monograph by Lin [3] and reviews by Sirignano and Mehring [4]. It is well established that the forces acting on a liquid gas interface including surface tension, pressure, inertia force, centrifugal force and viscous force result in the growth of disturbances that lead to sheet or jet breakup[2].

LINEAR INSTABILITY ANALYSIS

2.1. Model assumptions

The stability model considers a swirling inviscid annular liquid sheet subject swirling airstreams. Gas phases are assumed to be inviscid and incompressible.

The basic flow velocities for liquid, inner gas and outer gas are assumed to be $(U_l, 0, A_l/r)$, $(U_i, 0, \Omega r)$, $(U_o, 0, A_o/r)$ respectively and $A_o, A_l\ (m^2/s)$ are Vortex Strength and $\Omega\ (1/s)$ is Angular velocity.

Due to the swirling coaxial flow effect, centrifugal forces act on the annular liquid sheet. Additionally, inner and outer pressure forces are another type of forces acting on the inner and outer interfaces respectively. Furthermore, the liquid surface tension forces play an important effect on preventing the formation of a new surface. The sum of these forces determines weather the annular liquid sheet is going to breakup or remains stable [2].

2.2. Linearized disturbance equations

The governing equations for inviscid annular fluid flows are the continuity and Navier–

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Stokes equations that in cylindrical coordinate system are:
Continuity equation:

$$\frac{\partial U}{\partial x}+\frac{V}{r}+\frac{\partial V}{\partial r}+\frac{1}{r}\frac{\partial W}{\partial \theta}=0 \qquad (2.1)$$

Momentum equations:

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial r}+\frac{1}{r}W\frac{\partial U}{\partial \theta}=-\frac{1}{\rho}\frac{dP}{dx} \qquad (2.2)$$

$$\frac{\partial V}{\partial t}+U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial r}+\frac{1}{r}W\frac{\partial V}{\partial \theta}-\frac{W^2}{r}=-\frac{1}{\rho}\frac{dP}{dr} \qquad (2.3)$$

$$\frac{\partial W}{\partial t}+U\frac{\partial W}{\partial x}+V\frac{\partial W}{\partial r}+\frac{1}{r}W\frac{\partial W}{\partial \theta}+\frac{VW}{r}=-\frac{1}{\rho r}\frac{dP}{d\theta} \qquad (2.4)$$

The disturbances are assumed to have the forms:

$$(u,v,w,p')=(\hat{u}(r),\hat{v}(r),\hat{w}(r),\hat{p}'(r))e^{i(kx+n\theta-\omega t)} \qquad (2.5)$$

where ^ indicates the disturbance amplitude which is a function of r only. For the temporal analysis, the wave number k and n are real while frequency ω is complex. The imaginary part of ω reflects the growth rate of the disturbance. The displacement disturbances at the inner and outer interfaces are:

$$\eta_j(x,\theta,t)=\hat{\eta}_j e^{i(kx+n\theta-\omega t)}; \quad j=i,o \qquad (2.6)$$

To obtain the linearized disturbance equations, let:

$$U=\overline{U}+u,\ V=v,\ W=\overline{W}+w,\ p=P+p' \qquad (2.7)$$

Where the over bar represents the assumed mean flow quantities and the prime indicates disturbance.
The linearized disturbed equations for the liquid phase are written in vector form as:

$$\nabla.\vec{v}=0 \qquad (2.8)$$

$$\left(\frac{\partial}{\partial t}+\frac{A}{r^2}\frac{\partial}{\partial \theta}+U_l\frac{\partial}{\partial x}\right)\vec{v}=-\frac{1}{\rho}\nabla p_l' \qquad (2.9)$$

The linearized disturbed equations for the inner and outer air are written in component form as
Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{v}{r}+\frac{\partial v}{\partial r}+\frac{1}{r}\frac{\partial w}{\partial \theta}=0 \qquad (210)$$

Momentum equations:

$$\frac{\partial u}{\partial t}+U_j\frac{\partial u}{\partial x}+\frac{W_j}{r}\frac{\partial u}{\partial \theta}=-\frac{1}{\rho_j}\frac{\partial p'_j}{\partial x} \qquad (2.11)$$

$$\frac{\partial v}{\partial t}+U_j\frac{\partial v}{\partial x}+\frac{W_j}{r}\frac{\partial v}{\partial \theta}-\frac{2W_j w}{r}=-\frac{1}{\rho_j}\frac{\partial p'_j}{\partial r} \qquad (2.12)$$

$$\frac{\partial w}{\partial t}+U_j\frac{\partial w}{\partial x}+v\frac{\partial W_j}{\partial r}+\frac{W_j}{r}\frac{\partial w}{\partial \theta}+\frac{W_j v}{r}=-\frac{1}{\rho_j r}\frac{\partial p'_j}{\partial \theta} \qquad (2.13)$$

where, $j=i,o$ *and* $We_i=r\Omega, W_o=A_o/r$

Boundary conditions must be applied at the liquid interface. The first boundary condition is the kinematics condition can be expressed for the inner interface at $r=R_a$ as:

$$v_i=\frac{D\eta_i}{Dt}=\frac{\partial \eta_i}{\partial t}+U_i\frac{\partial \eta_i}{\partial x}+\Omega\frac{\partial \eta_i}{\partial \theta} \qquad at \quad r=R_a \qquad (2.14)$$

$$v_l=\frac{D\eta_i}{Dt}=\frac{\partial \eta_i}{\partial t}+U_l\frac{\partial \eta_i}{\partial x}+\frac{A_l}{r^2}\frac{\partial \eta_i}{\partial \theta} \qquad at \quad r=R_a \qquad (2.15)$$

And for the outer interface at $r=R_b$ as:

$$v_o=\frac{D\eta_o}{Dt}=\frac{\partial \eta_o}{\partial t}+U_i\frac{\partial \eta_o}{\partial x}+\Omega\frac{\partial \eta_o}{\partial \theta} \qquad at \quad r=R_b \qquad (2.16)$$

$$v_l=\frac{D\eta_o}{Dt}=\frac{\partial \eta_o}{\partial t}+U_l\frac{\partial \eta_o}{\partial x}+\frac{A_l}{r^2}\frac{\partial \eta_o}{\partial \theta} \qquad at \quad r=R_b \qquad (2.17)$$

The second boundary condition considers the balance between the surface stresses on both sides of the liquid-gas interface, including the pressure jump across the interface due to surface tension and viscous forces. This boundary condition is known as the dynamic boundary condition and is given by:

$$p_l'-p_i'=\sigma\left(\frac{\eta_i}{R_a^2}+\frac{\partial^2\eta_i}{\partial x^2}+\frac{1}{R_a^2}\frac{\partial^2\eta_i}{\partial \theta^2}\right)+\rho_i\Omega^2R_a\eta_i-\frac{\rho_l A_l^2\eta_i}{R_a^3}$$

$$at \ r=R_a \qquad (2.18)$$

$$p_l'-p_o'=-\sigma\left(\frac{\eta_o}{R_b^2}+\frac{\partial^2\eta_o}{\partial x^2}+\frac{1}{R_b^2}\frac{\partial^2\eta_o}{\partial \theta^2}\right)+\frac{\rho_o A_o^2\eta_o}{R_b^3}-\frac{\rho_l A_l^2\eta_o}{R_b^3}$$

$$at \ r=R_b \qquad (2.19)$$

The pressure disturbances can be obtained inside the liquid sheet, inner air and outer air after solving the equations to form of following respectively:

$$p_l' = \frac{\rho_l}{k}\left(\omega - kU_l - \frac{nA_l}{r^2}\right)\left(c_1 I_n(kr) + c_2 K_n(kr)\right)e^{i(kx+n\theta-\omega t)}$$

(2.20)

$$p_i' = \frac{\rho_i\left[(-\omega + kU_i + n\Omega)^2 - 4\Omega^2\right](-\omega + kU_i + n\Omega)\hat{\eta}_i I_n(k_1 r)e^{i(kx+n\theta-\omega t)}}{\left[(-\omega + kU_i + n\Omega)k_1 I_n'(k_1 R_a) + \frac{2n\Omega}{R_a} I_n(k_1 R_a)\right]}$$

(2.21)

$$p_o' = \frac{\rho_o}{k}\left(\omega - kU_o - \frac{nA_o}{R_b^2}\right)\left(\omega - kU_o - \frac{nA_o}{r^2}\right)\hat{\eta}_o \frac{K_n(kr)}{K_n'(kR_b)} e^{i(kx+n\theta-\omega t)}$$

(2.22)

where, $I_n(kr), K_n(kr)$ are the n^{th} order modified Bessel Function of first and second kind respectively.

The dispersion equation is obtained by substituting the pressure disturbances inside the liquid and gas phases into the dynamic boundary conditions at the two interfaces. The fourth order dispersion equation is obtained and can be written in below form. The Final Dispersion Equation was solved numerically using the secant method.

$$c_7\varpi^4 + c_8\varpi^3 + c_9\varpi^2 + c_{10}\varpi + c_{11} + $$

$$-\frac{g_i\left(-c_1\varpi^4 + c_{12}\varpi^3 + c_{13}\varpi^2 + c_{14}\varpi + c_{15}\right)}{c_{16} I_n(h\bar{k}_1) + (c_4 - \varpi)\sqrt{1 - \frac{c_5}{(c_4 - \varpi)^2}}\, I_n'(h\bar{k}_1)}(c_4 - \varpi) I_n(h\bar{k}_1) = 0$$

3. RESULTS AND CONCLUSIONS

For each pair of dimensionless parameters $(n, \bar{k})$, we solve fourth order non-linear dispersion equation for the root $\varpi = \omega R_b / U_l$ with maximum imaginary part that represents the maximum growth rate of the disturbance. The results show that the highest unstable wave number is achieved by increasing the inner air swirl Weber number when it is applied on the second annular liquid sheet helical mode (n=2) utilizing the co-inner and counter-outer swirl orientation as shown in Figure 1.

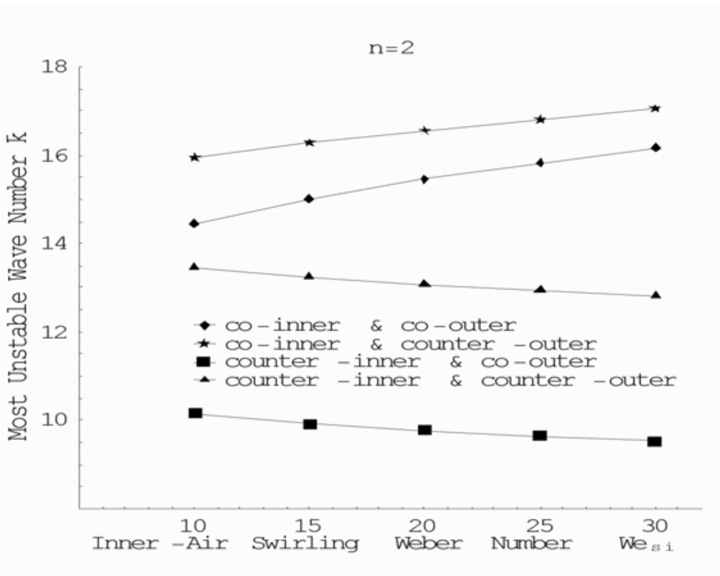

Figure 1. *Effect of the inner air swirling on the disintegration of the second helical annular liquid sheet at* $We_l = 35, We_i = 25, We_o = 15, We_s = 0.01, We_{so} = 10, h = 0.9$

The shortest breakup length is achieved by increasing the inner air swirl Weber number in the co-inner and co-outer swirl orientation.

The growth rate can be related to the breakup length of the liquid sheet. Higher growth rate indicates shorter breakup length.

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