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Original Contribution

APPLICATION OF SPLINE FUNCTIONS AND FUZZY NEURAL NETWORKS IN RELIABILITY EVALUATION OF RISK TECHNICAL OBJECTS

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ABSTRACT

The use of spline functions in diagnostic and prediction tasks of the technical systems state is examined. The proposed methods are elaborated with supposition for a closed and limited examined region in the multitude of conditions of the phase space. In situations with multidimensional and repeated failures, intrinsic to the technical systems (automobiles, aircrafts, railway, sea and river transport, chemical installations, munitions and other), the use of these methods demands additional examinations, as shown in the present article.

Key words: spline functions; risk technical systems

INTRODUCTION

The reliability estimate of Risk Technical Systems (RTS) using spline functions is connected with an analysis of the problem for existing and construction of these functions. The issue to be addressed is the existence and unity of a multidimensional spline in the frontier areas of use and the methods for its design. This question is examined in the present article on the basis of the determination of splines in accordance with references [1; 2].

PRESENTATION

We initiate the following indications: X, Y, Z - Hilbert space; $A : X \to Z$ and $T : X \to Y$, where $N(A)$ and $N(T)$ are the kernels of the operators. On the basis of both formulations and the initiated indications we define the relevant interpolation and polishing spline. If the condition $A^{-1}(z) = \varnothing, z \in Z$ is fulfilled, then interpolation spline σ will be every submultitude of $\sigma \in X$, for which is executed:

(1) $$\|T.\sigma\|_Y^2 = \min_{x \in A^{-1}(z)} \|T_x\|_Y^2 .$$

Let's consider the multitude $\alpha \in R^+$ and to construct a function in X as follows:

(2) $$\Phi_\alpha(x) = \alpha.\|T.x\|_Y^2 + \|A.x - z\|_z^2 .$$

The element $\sigma_\alpha \in X$ is called polishing spline, if the equality (3) is executed:

(3) $$\Phi_\alpha(\sigma_\alpha) = \min_{x \in X} \Phi_\alpha(x) .$$

The shown variables are interpreted physically in the following way: A - an operator, defining the structure of the made measurements (periodicity, dependence of the parameters, precision of the measurements); Z - a vector of the measured parameters; X - a concrete functional space, including functions of such degree of fluency, which is defined by a - priory information about the character of the processes, carrying out in the RTS; T - an operator defining the criterion of correspondence with the modelling function of the examined process. For investigation of the existing and uniqueness of n - measured spline for prediction of the RTS state, the following theorem is formulated, which is to be proved in this article.

THEOREM

If the multitude $TN(A)$ is closed in Y and $N(A) \cap N(T) = O_x$, then interpolation

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splain $\sigma_\alpha \in X$, appears as a decision of the tasks $A\sigma = z$ and $\|T\sigma\|_Y^2 = \min$, exists and it is defined unique for every $z \in Z$, satisfying the condition $A^{-1}(Z) \neq \varnothing$.

Evidence: Let the condition $A^{-1}(Z) \neq \varnothing$ be done, which follows from the physical considerations for RTS functionality. Let's examine the element $x \in A^{-1}(Z)$. From the definition for nucleus and from the attribute for linearity for the operator A it can be seen that:

(4) $A^{-1}(Z) = x + N(A)$.

Consequently, from the geometrical considerations we have a replacing of $A^{-1}(Z)$ of the linear space of the vector, X, i.e. $A^{-1}(Z)$ is an affinity variety in space X. Let's examine the expression:

(5) $TA^{-1}(Z) = TX + TN(A)$.

Because it is a linear operator, then $TA^{-1}(Z)$ is also an affinity variety, but now by Y and by condition from theorem $TA^{-1}(Z)$ is closed in Y. It is follows that the expression $\|T\Theta\|_Y^2 = \min$ is equivalent to minimum of the interval from O_Y to the affinity variety $TA^{-1}(Z)$. Moreover, in this closeness exists an element $f \in TA^{-1}(Z)$ assuring such a minimum. Due to condition (1) it follows, that $T\Theta = f$, where T is a linear operator, then it follows that Θ exists. Now it follows a proof for the uniqueness of the element, f. Let two points f_1 and f_2, exist so that $\|f_1\|_Y = \|f_2\|_Y$. Hence, it follows:

(6) $f_1 \in TA^{-1}(Z)$, $f_2 \in TA^{-1}(Z)$.

Because the operators T and A are linear, then follows:

(7) $\dfrac{f_1 + f_2}{2} \in TA^{-1}(Z)$.

From equations (6) and (7), using the quality linearity of the standard the equations are written:

(8)
$$\|f_1\|_Y \le \frac{\|f_1 + f_2\|_Y}{2} \le 0{,}5\|f_1\|_Y + 0{,}5\|f_2\|_Y = \|f_1\|_Y$$

(9)
$$\|f_2\|_Y \le \frac{\|f_1 + f_2\|_Y}{2} \le 0{,}5\|f_1\|_Y + 0{,}5\|f_2\|_Y = \|f_2\|_Y$$

We summarize member by member inequalities (7 ÷ 8), where we get:

(10)
$$\|f_1\|_Y + \|f_2\|_Y \le \|f_1 + f_2\|_Y \le \|f_1\|_Y + \|f_2\|_Y$$

From inequality (10) we get:

(11) $\|f_1 + f_2\|_Y = \|f_1\|_Y + \|f_2\|_Y$.

We raise the left and the right part of (11) to second degree and we get:

(12)
$$\|f_1\|_Y^2 + 2(f_1, f_2)_Y + \|f_2\|_Y^2 = \|f_1\|_Y^2 + 2\|f_1\|_Y.\|f_2\|_Y + \|f_2\|_Y^2$$

From equation (12) follows:

(13)
$$(f_1, f_2)_Y = \|f_1\|_Y.\|f_2\|_Y = \|f_1\|_Y^2 = \|f_2\|_Y^2 = (f_1, f_2)_Y = (f_2, f_2)$$

Therefore: $f_1 \equiv f_2$.

Now it can be shown that, for a given f, exists only one splines Θ, for which is done $\Theta \in A^{-1}(Z)$ and $T\Theta = f$. We suppose that splines Θ_1 and Θ_2 exist. It follows that $T\sigma_1 = f$, $T\Theta_2 = f$ and $A\Theta_1 = Z$, $A\Theta_2 = Z$. Let us consider $g = \Theta_2 - \Theta_1$. While change to the operators T and A, due to their linearity we get $Tg = T\sigma_2 - T\sigma_1 = f - f = O_Y$.

Similarly we get $Ag = O_Y$.

Hence, $g \in N(T) \cap N(A)$. From the conditions of the theorem it follows that $N(T) \cap N(A) = O_x$, therefore, $g = O_x$ or $\Theta_1 = \Theta_2$, which can be proved.

Now we prove the existence and the uniqueness of the polishing splines. We put (2) on the right part of (3), and we get

(14)
$$\Phi_\alpha(\Theta_\alpha) = \min\left[\alpha\|Tx\|_Y^2 + \|Ax - Z\|_z^2\right] =$$
$$= \alpha \min\|Tx\|_Y^2 + \|Ax - Z\|_z^2.$$

Because α and $\|Ax - Z\|_z^2$ are constant quantities, it follows, that the proof for existing and uniqueness of the polishing splain reduces to the evidence of the existence and uniqueness of the expression $\min\|Tx\|_Y^2$, which was already proved. Now let us examine the behaviour of RTS near to the limit of the area, for which are defined the peculiarity of design of the splines.

1. Self – removing the design failures

At border values of the parameters of RTS ε_i, equal to a and 0, must be near to the avalanche rise out of the permit. However, the area of all uninterrupted functions with values in the interval $[0, a]$ does not describe fully the examined process (**Figure 1).**

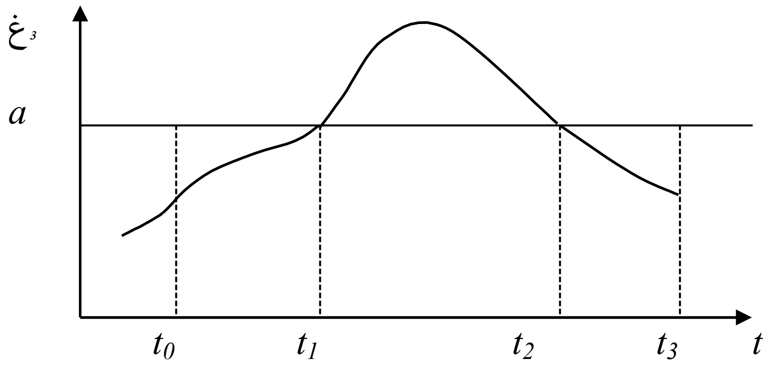

Figure 1: *Jump of the values of the parameter ξ_i of RTS out of the admissible borders of the Producer*

In the whole multitude of splines $I([0, a])$, located in the interval $[0, a]$, and by the theorem for existence and uniqueness, proved above, are found splines $g(t), h(t),$ so that the following can be fulfilled:

(15)

$$a)\ \{g(t), h(t)\} \in I([0, a])\ \forall t \in [t_0, t_3],$$

$$б)\ \|g(t) - \xi_i(t)\| \to 0, t \in [t_0, t_1] \mathrm{Y} [t_2, t_3],\ bI([0, a]),$$

$$в)\ h(t) \exists \& ! \bar{U}, \|\xi_i(t) - g(t).h(t)\| \to 0, t \in [t_1, t_2], bI([0, a]).$$

For the design of the modelling function of the process of examination, by splines in case, when $[t_1, t_2] \to 0$ it is necessary for an examination of the possibility for their design in the field of functions. Then, the conditions in (15) define the region of used splines $I[0, a]$.

2. Alternate failures

In work conditions, which define the limit values of the parameters ξ of RTS, parameter changes beyond their admissible value (alternate failures) come, for small time intervals (**Figure 2).**

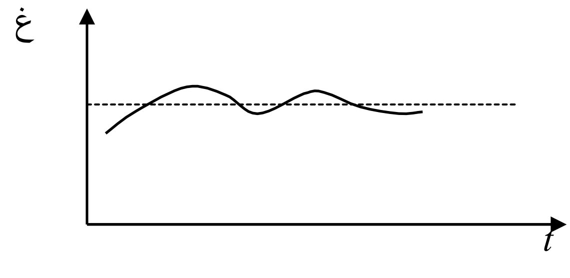

Figure 2. *Small deflections of relevant parameter of RTS beyond its admissible value*

For the modelling of the process of alternate failures, using splines functions it is necessary to prove that in the multitude of splines $I[0, a]$ for every group of three functions are the operations: multiplication, collecting, commutative ness and associative ness; so it is possible to have a work in circle or semicircle of splines describing the multitude of real number R^n.

That is why, without being violated the community of the ratiocinations, a task for design of polynomial functions in arbitrary areas is admissible. For such purpose, there must be the task for design of splines in circle or in field. The use of operations with splines over field of circle is necessary, due to the fact that design of functions, modelling the state of RTS can be done, by algebraic operations over the numbers (estimates of parameters) with different dimensions.

It is recognisable that the multitude R of a few elements is called circle, where are defined (possible) two operations-collecting and multiplication, together with the operations commutative ness and associative ness, connected with the laws of the distributives (because the collecting is connected with the opposite operation-deduction) [1].

The multitude of all spline, defined for $X \in R^n$, will be a circle, if they can do the operations of collecting and multiplication and the qualities commutative ness, associative ness and distributiveness. If the quality distributiveness is not applicable, than the multitude F will be semicircle of spline, including the multitude of real numbers R^n.

A circle of spline forming an area F, and including R^n will be field, if some zeros are contained (of course in them must be applicable the operation of partition) [2]. For all elements $f(x), g(x)$ of the area F, for which $f(x) \neq 0$, exists a spline function $h(x) \in F$, $f(x).h(x) = g(x)$, and in such a time the element $h(x)$ will be unique.

Obvious from these definitions follow the following conclusions:

- -for arbitrarily basic field F could be used the theory of the linear dependent vectors, the theory for solving of systems with linear equations (but only in case of an endless field F), and also a matrix algebra;
- for an arbitrary field F, could be used the theory of the linear areas and their linear transformations.

3. Mathematical models of failures

3.1. Model of self-removing failures

Self-removing failure from the point of view of the analysis of the parameters of RTS, is called every violation of the admissible values of the parameters during casual interval of time (much less than the technical resource to main repair of RTS put by the producer) [3, 4]. The missing of information for the real value of the parameters of RTS in the frames of the admissible boards, must be examined as a question, connected with the use of splines for the prediction of the technical state. For that purpose it is examined in the design of spline-function from the point of view for minimization of an integral function chosen for a criterion (in the particular case for $\Omega \subset R^n$ and third degree of the spline the above property is the minimum rate of Holliday) [5].

The minimum of the multiple integral must be found:

(16)
$$I(U) = \int_D \ldots \int F\left(x, y, U, U_x', U_y'\right).dx.dy,$$

$x \in R, y \in R^n, D \subset R^n$,

where: $U = \varphi(S)$ of Γ, and $\Gamma = \partial D, S \in R^n$, $\varphi(S)$ is a function describing the behaviour of the process around the limit of the area.

If U^* is the exact solution of that task, then with $I(U^*) = m$ could be noted the value of the minimum. If it is possible in the field D to be built a function, reaching the minimum value $\overline{U}(x, y)$, which satisfies (16), and $I(\overline{U})$ is near to m, then $\overline{U}$ will be a passable decision of the task.

If it is possible to minimize the sequence $\overline{U}_n$, i.e. the sequence from functions satisfying (16) for which $I(\overline{U}_n) \to m$, then a search for a solution in the field of spline is possible. It then becomes possible to examine a family splain function:

(17) $U = \Phi(x, y, a_1, a_2, \ldots, a_n)$,

where: $a_1, a_2, \ldots, a_n$ are arguments, characterizing numbers of the approximation function and depending on some parameters.

Then, for $\forall a_i$, $U = \varphi(S)$ is done. After substitution of formula (17) in (16), we get:

(18) $I = I(a_1, a_2, \ldots, a_n)$.

Equation (18) is minimized from the condition:

(19) $\partial I / \partial a_i = 0 \ \forall_i$.

After solution of the system of equations (19) we get values $a_i, i = \overline{1,..,n}$, for which,

(20) $I(a_1, a_2, \ldots, a_n) = \min$.

It is chosen from the family functions (20), such for which replays to the value $a_i, i = \overline{1, n}$, where we get the searching model of a self-removing failure:

(21) $\overline{U}(x,p)=\Phi(x,y,\overline{a}_1,\overline{a}_2,...,\overline{a}_n)$.

We must mention that the process of searching of that function is comparatively simple, except in the cases of work over a field (area), because all possible actions over the elements which are in them must be done. From what has been said above it follows, that the examined field of splines in the section of breaking of the function from **Fig.1** (coming out of the informative parameter of the permit), allows the same to be used for prediction of the technical state in the condition of self- removing failures.

3.2. Model of alternate failures

For design of that model it is necessary to examine the application of splines for prediction of the technical state in RTS. For that purpose we examine the conditions where our developed method could be used for design of splines in the circle D. These families of splines functions are hereby examined:

(22)
$$U_n=\Phi_n(x,a_1,a_2,...,a_n),\ (n=1,2,...)$$

where each of them is wider than the one before in the circle D as a result of adding extra parameters. Let $\overline{U}_n$ be a function, defining the least value of $I(U)$ defined by (16), in comparison with all other functions of the n family. Because of addition of parameters, the class of admissible functions is widening, then $I(\overline{U}_1)\geq I(\overline{U}_2)\geq ...$.

It could be defined that the condition, for which the succession of functions $\overline{U}_1,\overline{U}_2,...$ is minimum, i.e. the sequence $I(\overline{U}_1), I(\overline{U}_2)$ is striving for the minimum, is defined by:

(23) $\lim I(U_n)=I(U^*)=m$.

Sufficient condition for fulfilment of (23) is the relative completeness of the family splines functions from (22). Then the equality will be:

(24)
$$U_n^*(x)=\Phi_n(x_1,a_1^*,a_2^*,...,a_n^*),\ (n=1,2,...)$$

and the inequalities

$$|U_n^*-U|<\xi;\quad \left|\frac{\partial U_n^*}{\partial x}-\frac{\partial U}{\partial x}\right|<\xi$$.

From (22) and (23), it follows that every function could be very precisely described together with its private derivatives, by splain functions from the corresponding family in the circle D. That condition is sufficient for the requirement of the order $I(\overline{U}_1), I(\overline{U}_2)$ to strive for minimum. Actually if the condition for fulfilment is done, then using the solution of the task from (16), it is possible to have a selection of an appropriate splines function from the n- family for a model of alternate failures:

(25) $U_n^*(x)=\Phi_n(x,a_1^*,a_2^*,...,a_n^*)$,

so that we have the inequality:

(26)
$$\|U_n^*-U^*\|<\varepsilon,\qquad \left\|\frac{\partial U_n^*}{\partial x}-\frac{\partial U^*}{\partial x}\right\|<\xi$$.

From the execution of (26) and from the non-interruption of F, follows that the difference:

(27)
$$F\left(x,U_n^*,\frac{\partial U_n^*}{\partial x}\right)-F\left(x,U^*,\frac{\partial U^*}{\partial x}\right)$$,

would be possibly least in the circle D. And the difference $I(U_n^*)-I(U^*)$ defined also from the next equation:

(28)
$$I(U_n^*)-I(U^*)=\iint\left[F\left(x,U_n^*,\frac{\partial U_n^*}{\partial x}\right)-F\left(x,U^*,\frac{\partial U^*}{\partial x}\right)\right].dx;\ \xi>0$$

would also be possibly least. Then $\xi'\in R,\ \xi'>0$.

From (28), it follows that U_n^* is one of the functions of such family. Then this condition must always be executed as:

(29) $I(\overline{U}_n)\leq I(U_n^*)$

Or $I(U^*)\leq I(\overline{U}_n)\leq I(U_n^*)<I(U^*)+\xi'$.

Because ξ' is as least as possible, then the equation which is to be proved is as follows:

(30) $I(U^*)=I(\overline{U}_n)$.

4. RTS diagnosis with fuzzy neural networks and adaptive algorithm for self-organization

The adaptive algorithm for a self-organization of a fuzzy neural network to the moment is formulated only for a Gauss function with the usage of a Wang-Mendel summary model [6, 7]. This model helps to determine the following: the quantity of the centres and their location up parts, correspond to the condition (multitude of the vectors x_i) and conclusions (multitude of the scalars expectance values of d_i). This algorithm could be written in the following way:

4.1. At the start with the first two couples of data (x_1, d_1) the first cluster is made with centre $C_1 = x_1$. It is accepted, that $W_1 = d_1$ and the power of the multitude is $L_1 = 1$. Let r marks the limit of Euclidian distance between the vector x and a centre C_1, when the data will be determined as belonging to the made cluster. For preserving the community of the determination it is accepted that at the start of the training exists M clusters with centres $C_1, C_2, ..., C_M$ and their relative values W_i and $L_i \; (i = 1, 2, ..., M)$.

4.2. After reading of the k-ti learning couple (x_k, d_k) the distance between the vectors x_k and all existing centres $\|x_k - C_l\|$ for $l = 1, 2, ...M.$ is calculated. We permit, that the neighbouring centre is C_{l_k}. In that case according to the values of $\|x_k - C_{l_k}\|$ could arise one of the two following situations:

If $\|x_k - C_{l_k}\| > r$, then a new cluster $C_{M+1} = x_k$ is made, so we have $W_{M+1}(k) = d_k$; $L_{M+1}(k) = 1$ The parameters made to that cluster does not change, i.e. we have $W_l(k) = W_l(k-1)$; $L_l(k) = L_l(k-1)$ for $l = 1, 2, ...M.$ The quantity of the clusters M increases with 1, i.e. $(M \leftarrow M + 1)$;

If $\|x_k - C_{l_k}\| \le r$, then the data is included in l_k - th cluster, whose parameters follows to be specified in correspondence with the formulas [9]:

(31) $W_{l_k}(k) = W_{l_k}(k-1) + d_k$,

(32) $L_{l_k}(k) = L_{l_k}(k-1) + 1$,

(33)
$$C_{l_k}(k) = \frac{C_{l_k}(k-1).L_{l_k}(k-1) + x_k}{L_{l_k}(k)}.$$

In another version the algorithm is fixed by the position of the centres C_{l_k} after initialisation and their coordination already does not change. In many cases that approach improves the results from the adaptation.

4.3. After specifying the parameters of the fuzzy system, the approximate function of the input data are determined in case:

(34)
$$\hat{f}(x) = \frac{\sum_{l=1}^{M} W_l(k).\exp\left(-\frac{\|x - C_l(k)\|^2}{\sigma^2}\right)}{\sum_{l=1}^{M} L_l(k).\exp\left(-\frac{\|x - C_l(k)\|^2}{\sigma^2}\right)}$$

When the rest of clusters don't change, i.e. at $l \ne l_k$ these formulas are valid:
$W_l(k) = W_l(k-1)$
$L_l(k) = L_l(k-1)$
and $C_l(k) = C_l(k-1)$ for $l = 1, 2, ...M.$
At repetition of the enumerated stages of the algorithm to $k = p$ with modification of every value of M, the space of data are divided into M clusters. Then the power of each of them is determined as $L_l = L_l(k)$, but for centres as $C_l = C_l(k)$. The value of function d, determined as $W_l = W_l(k)$.

That algorithm is named self-organized, as far as the differentiation of the data space is necessary for diagnosis of the RTS is independent. The diagnosis is done without a human participation and in correspondence with determined values of a limit r. In a small value of r the quantity of the cluster increases. As a result of that the approximation of the data gets more precise. But that could be achieved for more complex function. If the value r is very big, then the calculated power is smaller. That provokes increase of the uncertainty of approximation. At a selection of the optimal value at the limit of diagnosis, r should be made a compromise between the certainty of reflection of the condition of the RTS and the calculated complexity. As a norm, the optimum value of r is selected with the method of the "tests and errors" [2, 8] and the usage of a calculated experimental results shown in literature [7, 9, 10].

On **Figure 3 and Figure 4** are shown the results from the approximation of the function $f(x)$ which is an example for analytical model of the condition of RTS. That function has a tape:

$$f(x)=0{,}1.\sin(0{,}2\pi x)+0{,}2.\sin(0{,}3\pi x)+0{,}6.\sin(0{,}9\pi x)+$$
$$+1{,}1.\sin(1{,}9\pi x)+2{,}3.\sin(3{,}7\pi x).$$

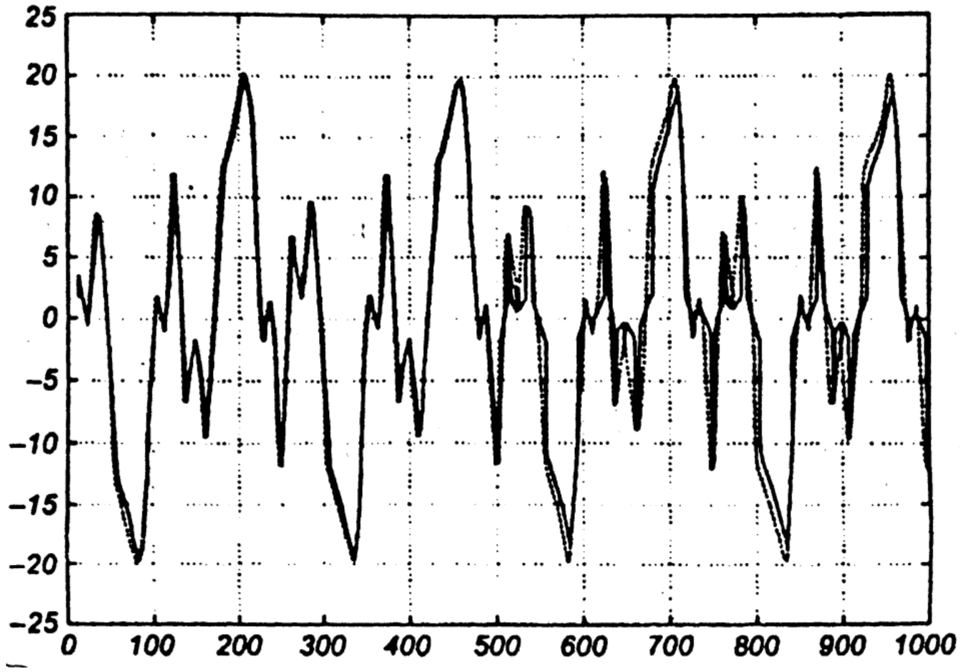

Figure 3. *An example for diagnostic of the RTS by* $r=0{,}2$

The diagnosis of $f(x)$ is made, by fuzzy neural network with self-organized, working with adaptive algorithm for learning. It has level for diagnosis $r=0{,}2$ at **Figure 3** and $r=0{,}05$ by **Figure 4.**

On **Figure 3** and **Figure 4** the punctuation line marked the expectance values, and with the non-determined line are marked the current values generated of the neural model of RTS. The algorithm has selected the quantity of the neurons, corresponding to the fixed level of the diagnosis r alone. At the first case the fixed by the algorithm quantity of the neurons is 12, and in second case is 19.

For learning only the first 500 realizations of the signal are used. The remaining 500 values are used only for test. The algorithm of the self-organized of the fuzzy network allows at the same time to be defined the parameters of the network and its structure (the value of the neurons in the hidden layer).

His realization is similar to the Wang-Mendel model [6], which could separate the centres C_l. These centres are respective to the multitude of the vectors x and the coefficients W_l, connected with the situation at the centres by the succession from the set functions $\{d\}$. In connection with the conglomerate character of the forming of the parameters W_l determined by formula (31), in the denominator of (34), summing up must be done using L_l. They reflect quantity of the specific parameters of the concrete data groups, i.e. the dimension of the cluster.

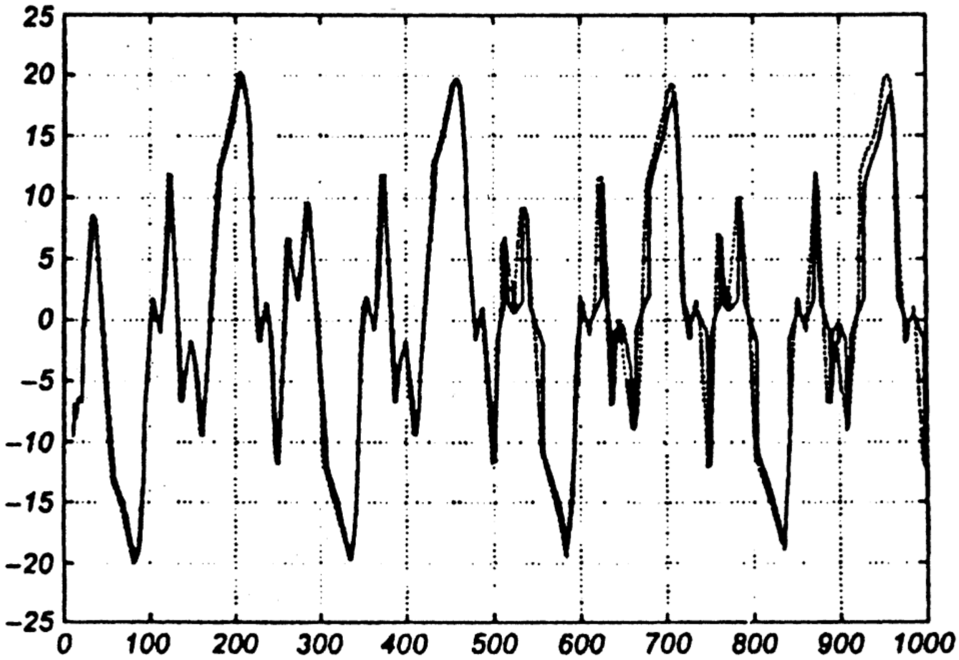

Figure 4. *An example for diagnostic of the RTS by* $r = 0,05$

CONCLUSION

1. From the suggested and proved theorem for interpolation by spline functions and from the created models of the self-removing and alternated failures follows the possibility for one authentic realization of prognostication of the technical condition of the risk technical systems.
2. The approach of the spline-functions is applicable and in modelling the conduct of risk technical systems and also for valuation of hidden despairs.
3. The use of self organized algorithm for diagnosis of risk technical systems is possible, when the partition of the data space necessary for diagnosis runs independently. The diagnosis is accomplished without the participation of the person and in correspondence with the set valuations of the threshold of diagnosis.

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